7.3 Notes

7.3: Nonterminating Decimals

Fact: The fraction $\frac{a}{b}$ is equivalent to the problem $a \div b$.

Example: Convert $\frac{3}{4}$ to a decimal.

Example: Try to write $\frac{1}{3}$ as a decimal number using division.

Definition: A repeating decimal is a decimal whose digits repeat every fixed number of digits.

Example: Convert the following fractions to decimals.

(a) $\frac{5}{11}$

(b) $\frac{2}{9}$

(c) $\frac{1}{7}$

Example: Convert the following repeating decimals to fractions.

(a) $0.\overline{3}$

(b) $0.\overline{09}$

(c) $0.1\overline{6}$

(d) $0.0\overline{45}$

1

(c) $0.\overline{27}$

(c) $0.2\overline{27}$

2

(d) $0.\overline{126}$

(f) $0.\overline{9}$

Ordering Decimals:

To determine which of two decimals is larger, we consider their fraction representation.

Example: Show that $1.051 > 1.0495$.

In summary, to order decimals, we consider the following:

1. If the whole number is larger, then the decimal is larger.
2. If the whole number is smaller, then the decimal is smaller.
3. If the whole numbers are equal, then we consider each decimal place (adding zeroes if necessary) until we reach the first decimal place where the digits are different. Whichever number has the larger digit is the larger number.
4. If the whole numbers and all the digits are the same, the decimals are equal.

Example: Fill in the blanks with $<$, $>$, or $=$.

1.234 ___ 0.345

5.62 ___ 5.621

2.207 ___ 2.211

$1.\overline{45}$ ___ $1.45\overline{4}$

$1.\overline{36}$ ___ 1.363

Notice that everything we've done so far with decimals were converted from fractions. Are there decimals that can't be written as fractions?

Definition: The irrational numbers, denoted I, are the real numbers that are not rational numbers.

What would the irrational numbers look like as decimals?